

Bouncing cosmological solutions due to the self-gravitational corrections and their stability

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Received: 20 May 2006 /

Published online: 30 June 2006 – © Springer-Verlag / Società Italiana di Fisica 2006

Abstract. In this paper we consider the bouncing braneworld scenario, in which the bulk is given by a five-dimensional AdS black hole spacetime with matter field confined in a 3D brane. Exploiting the CFT/FRW-cosmology relation, we consider the self-gravitational corrections to the first Friedmann-like equation which is the equation of the brane motion. The self-gravitational corrections act as a source of stiff matter contrary to standard FRW cosmology, where the charge of the black hole plays this role. Then we study the stability of solutions with respect to homogeneous and isotropic perturbations. Specifically, if we do not consider the self-gravitational corrections, the AdS black hole with zero ADM mass and open horizon is an attractor, while, if we consider the self-gravitational corrections, the AdS black hole with zero ADM mass and flat horizon is a repeller.

1 Introduction

Motivated by string/M theory, the AdS/CFT correspondence, and the hierarchy problem of particle physics, braneworld models have been studied actively in recent years [1–4]. In these models, our universe is realized as a boundary of a higher dimensional spacetime. In particular, a well studied example is when the bulk is an AdS space. In the cosmological context, embedding of a four-dimensional Friedmann–Robertson–Walker universe was also considered when the bulk is described by AdS or an AdS black hole [5, 6]. In the latter case, the mass of the black hole was found to effectively act as an energy density on the brane with the same equation of state of the radiation. Representing radiation as conformal matter and exploiting the AdS/CFT correspondence, the Cardy–Verlinde formula [7] for the entropy was found for the universe (see [8] for the entropy formula in the case of a dS black hole).

In either of the above cases, however, the cosmological evolution on the brane is modified at small scales. In particular, if the bulk space is taken to be an AdS black hole *with charge*, the universe can ‘bounce’ [9]. That is, the brane makes a smooth transition from a contracting phase to an expanding phase. From a four-dimensional point of view, singularity theorems [10] suggest that such a bounce cannot occur as long as certain energy conditions apply. Hence, a key ingredient in producing the bounce is the fact that the bulk geometry may contribute a negative energy density to the effective stress-energy on the brane [11]. At first sight these bouncing braneworlds are quite remark-

able, since they provide a context in which the evolution evades any cosmological singularities while the dynamics is still controlled by a simple (orthodox) effective action. In particular, it seems that one can perform reliable calculations without deliberation on the effects of quantum gravity or the details of the ultimate underlying theory. Hence, several authors [12–15] have pursued further developments for these bouncing braneworlds. However the authors of [16] have found that generically these cosmologies are in fact singular. In particular, they have shown that a bouncing brane must cross the Cauchy horizon in the bulk space. However, the latter surface is unstable when arbitrarily small excitations are introduced in the bulk spacetime.

Black hole thermodynamic quantities depend on the Hawking temperature via the usual thermodynamic relations. The Hawking temperature undergoes corrections from many sources: the quantum corrections [17–22], the self-gravitational corrections [23, 24], and the corrections due to the generalized uncertainty principle [25, 26]. Concerning the quantum process called the Hawking effect [27] much work has been done using a fixed background during the emission process. The idea of Keski-Vakkuri, Kraus and Wilczek (KKW) [23] is to view the black hole background as dynamical by treating the Hawking radiation as a tunnelling process. The energy conservation is the key to this description. The total (ADM) mass is kept fixed while the mass of the black hole under consideration decreases due to the emitted radiation. The effect of this modification gives rise to additional terms in the formulae concerning the known results for black holes [24]; a non-thermal partner to the thermal spectrum of the Hawking radiation shows up.

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In this paper we take into account corrections to the entropy of the five-dimensional Schwarzschild–anti de Sitter black hole (abbreviated to SAdS₅ in the sequel) that arise due to the self-gravitational effect. The self-gravitational correction acts as a source for stiff matter on the brane, whose equation of state is simply given by the pressure being equal to the energy density. Due to the self-gravitational corrections, a bouncing universe could arise [15]. Then we describe solutions of the bouncing braneworld theory and also determine their stability. To do this, we use a set of convenient phase-space variables similar to those introduced in [28, 29]. The critical points of the system of differential equations in the space of these variables describe interesting non-static solutions. A method for evaluating the eigenvalues of the critical points of the Friedmann and Bianchi models was introduced by Goliath and Ellis [29] and further used in the analysis by Campos and Sopena [28] of the Randall–Sundrum braneworld theory. These latter authors gave a complete description of stationary points in an appropriately chosen phase space of the cosmological setup and investigated their stability with respect to homogeneous and isotropic perturbations. The authors worked in the frames of the Randall–Sundrum braneworld theory without the scalar-curvature term in the action for the brane. Cosmological solutions and their stability with respect to homogeneous and isotropic perturbations in the braneworld model with the scalar-curvature term in the action for the brane have further been studied by Iakubovskiy and Shtanov [30].

2 Self-gravitational corrections to FRW equation

In asymptotic coordinates, the SAdS₅ black hole metric is

$$ds^2 = -F(r) dt^2 + \frac{1}{F(r)} dr^2 + r^2 d\Omega_{(3)}^2, \quad (1)$$

where

$$F(r) = 1 - \frac{\mu}{r^2} + r^2, \quad (2)$$

and we work in units where the AdS radius $l = 1$. The parameter μ is proportional to the ADM mass M of the black hole.

We now consider a four-dimensional brane in the SAdS₅ black hole background. This four-dimensional brane can be regarded as the boundary of the five-dimensional SAdS₅ bulk background. Let us first replace the radial coordinate r with a and so the line element (1) becomes

$$ds^2 = -F(a) dt^2 + \frac{1}{F(a)} da^2 + a^2 d\Omega_{(3)}^2. \quad (3)$$

It was shown that by reduction from the SAdS₅ background (3) and by imposing the condition

$$-F(a) \left(\frac{\partial t}{\partial \tau} \right)^2 + \frac{1}{F(a)} \left(\frac{\partial a}{\partial \tau} \right)^2 = -1, \quad (4)$$

where τ is a new time parameter, one obtains an FRW metric for the four-dimensional timelike brane,

$$ds_{(4)}^2 = -d\tau^2 + a^2(\tau) d\Omega_{(3)}^2. \quad (5)$$

Thus, the four-dimensional FRW equation describes the motion of the brane universe in the SAdS₅ background. It is easy to see that the matter on the brane can be regarded as radiation and, consequently, the field theory on the brane should be a CFT.

Within the context of context the AdS/CFT correspondence, Savonije and Verlinde studied the CFT/FRW-cosmology relation from the Randall–Sundrum type braneworld perspective [31]. They showed that the entropy formulas of the CFT coincide with the Friedmann equations when the brane crosses the black hole horizon.

In the timelike four-dimensional case

$$ds_{(4)}^2 = -d\tau^2 + a^2(\tau) d\Omega_{(3)}^2, \quad (6)$$

one of the identifications that supports the CFT/FRW-cosmology relation is

$$H^2 = \left(\frac{2G_4}{V} \right)^2 S^2, \quad (7)$$

where H is the Hubble parameter defined by $H = \frac{1}{a} \frac{da}{d\tau}$; V is the volume of the 3-sphere ($V = a^3 V_3$), and S is the entropy of the black hole. The four-dimensional Newton constant G_4 is related to the five-dimensional one G_5 by

$$G_4 = \frac{2}{l} G_5. \quad (8)$$

It was shown that at the moment that the four-dimensional timelike brane crosses the cosmological horizon, i.e. when $a = a_b$, the CFT entropy and the entropy of the SAdS₅ black hole are identical. The modified Hubble equation, i.e. the first Friedmann equation, takes the form

$$H^2 = \frac{-1}{a_b^2} + \frac{8\pi G_4}{3} \rho - \frac{8\pi G_4}{3} \left[\frac{4\pi G_4}{3} \frac{1}{a_b^2 V_3} \rho \right] \omega, \quad (9)$$

where the volume V is given by $a_b^3 V_3$, ρ is the energy density and ω is the energy of an emitted particle from the black hole. At this point it should be stressed that our analysis was up to now restricted to the spatially flat ($k = +1$) spacelike brane.

We will now extend the analysis. We therefore consider an arbitrary scale factor a and include a general k taking values $+1, 0, -1$ in order to describe, respectively, the elliptic, flat, and hyperbolic horizon geometry of the SAdS₅ bulk black hole. The modified Hubble equation is now given by

$$H^2 = \frac{-k}{a^2} + \frac{8\pi G_4}{3} \rho - \frac{8\pi G_4}{3} \left[\frac{4\pi G_4}{3} \frac{1}{a^2 V_3} \rho \right] \omega, \quad (10)$$

where the volume V is now given by $a^3 V_3$ since all quantities that appear in (10) are defined for an arbitrary scale factor a .

The first term in the right-hand side of (10) represents the curvature contribution to the brane motion. The second term can be regarded as the contribution from the radiation and it redshifts as a^{-4} . The last term in the right-hand side of (10) is the self-gravitational correction to the motion of a four-dimensional timelike brane moving in the five-dimensional Schwarzschild–anti de Sitter bulk background. Since this term goes like a^{-6} , it is obvious that it is dominant at early times of the brane evolution, while at late times the second term, i.e. the radiative matter term, dominates and thus the last term can be neglected. The sign of the last term is opposite with respect to the standard situation, and one may expect that this sign difference could have interesting cosmological consequences. Indeed, we will see that it is crucial in allowing a nonsingular transition between a contracting and an expanding evolution of the scale factor a .

3 Stability of the bouncing solutions

In this section, we describe the bouncing solutions of the braneworld theory under investigation and also determine their stability. To do this, we use a set of convenient phase-space variables similar to those introduced in [28, 29]. At first we rewrite (10) as

$$H^2 = \frac{-k}{a^2} + \frac{\varepsilon_3 M}{a^4} - \frac{\varepsilon_3^2 M \omega}{2a^6}, \tag{11}$$

where

$$\varepsilon_3 = \frac{16\pi G_5}{3V_3}. \tag{12}$$

Now, we introduce a notation similar to [28]:

$$\Omega_k = \frac{-k}{a^2 H^2} = \frac{-k}{\dot{a}^2}, \quad \Omega_M = \frac{\varepsilon_3 M}{a^4 H^2}, \quad \Omega_\omega = \frac{-\varepsilon_3^2 M \omega}{2a^6 H^2}, \tag{13}$$

and we work in the three-dimensional Ω -space $(\Omega_k, \Omega_M, \Omega_\omega)$.

In this space, the Ω parameters are not independent, since the Friedmann equation (11) reads

$$\Omega_k + \Omega_M + \Omega_\omega = 1. \tag{14}$$

Since these terms are non-negative they must belong to the interval $[0, 1]$, and hence the variables $\Omega = (\Omega_k, \Omega_M, \Omega_\omega)$ define a compact state space. Introducing the primed time derivative

$$\dots' = \frac{1}{H} \frac{d}{dt} \dots, \tag{15}$$

one obtains the system of first-order differential equations [28]

$$\begin{aligned} \Omega_k' &= 2q\Omega_k, \\ \Omega_M' &= 2(q-1)\Omega_M, \\ \Omega_\omega' &= 2(q-2)\Omega_\omega, \end{aligned} \tag{16}$$

where

$$q = \frac{-1}{H^2} \frac{\ddot{a}}{a} = \Omega_M + 2\Omega_\omega. \tag{17}$$

The behavior of this system of equations in the neighborhood of its stationary point is determined by the corresponding matrix of its linearization. The real parts of its eigenvalues tell us whether the corresponding cosmological solution is stable or unstable with respect to the homogeneous perturbations [30]. To begin with, we have to find the critical points of this dynamical system, which can be written in vector form as follows:

$$\Omega' = f(\Omega), \tag{18}$$

where f can be extracted from (16). The critical points, Ω^* , namely the points at which the system will stay provided it is initially there, are given by the condition

$$f(\Omega^*) = 0. \tag{19}$$

Their dynamical character is determined by the eigenvalues of the matrix

$$\left. \frac{\partial f}{\partial \Omega} \right|_{\Omega=\Omega^*}. \tag{20}$$

If the real part of the eigenvalues of a critical point is not zero, the point is said to be *hyperbolic* [28]. In this case, the dynamical character of the critical point is determined by the sign of the real part of the eigenvalues: If all of them are positive, the point is said to be a *repeller*, because arbitrarily small deviations from this point will move the system away from this state. If all of them are negative the point is called an *attractor* because if we move the system slightly from this point in an arbitrary way, it will return to it. Otherwise, we say the critical point is a *saddle* point.

We construct our models as follows.

(1) The model k , or $(\Omega_k, \Omega_M, \Omega_\omega) = (1, 0, 0)$. We have

$$q = 0, \tag{21}$$

and the eigenvalues are

$$\lambda_M = -2, \quad \lambda_\omega = -4. \tag{22}$$

(2) The model M , or $(\Omega_k, \Omega_M, \Omega_\omega) = (0, 1, 0)$. We have

$$q = 1, \tag{23}$$

and the eigenvalues are

$$\lambda_k = 2, \quad \lambda_\omega = -2. \tag{24}$$

(3) The model ω , or $(\Omega_k, \Omega_M, \Omega_\omega) = (0, 0, 1)$. We have

$$q = 2, \tag{25}$$

and the eigenvalues are

$$\lambda_k = 4, \quad \lambda_M = 2. \tag{26}$$

The dynamical system (16) has three hyperbolic critical points as follows.

i) The model k ($k = -1$),

$$M = \omega = 0, \quad a(t) = t,$$

with the critical point of *attractor* type.

ii) The model M ,

$$k = \omega = 0, \quad a(t) = (M\varepsilon_3)^{1/4}\sqrt{2t},$$

with the critical point of *saddle point* type.

iii) The model ω ,

$$k = M = 0, \quad a(t) = \left(\frac{3\sqrt{3}}{4}\varepsilon_3\omega t\right)^{1/3},$$

with the critical point of *repeller* type.

4 Conclusion

In this paper we have considered a four-dimensional time-like brane with non-zero energy density as the boundary of the SAdS₅ bulk background. Exploiting the CFT/FRW-cosmology relation, we have considered the self-gravitational corrections to the first Friedmann-like equation which is the equation of the brane motion. The additional term, that arises due to the semiclassical analysis, can be viewed as a stiff matter term where the self-gravitational corrections act as the source for it. This result is contrary to the standard analysis that regards the charge of a SAdS₅ bulk black hole as the source for stiff matter. Then we have studied bouncing cosmological solutions and their stability with respect to homogeneous and isotropic perturbations in a braneworld theory. The effects of the self-gravitational corrections of the five-dimensional black hole in the bulk have been considered. By including this effect in the analysis we have obtained three models with the critical points of an *attractor*, a *saddle point* and a *repeller*, respectively, and we have constructed the complete state space for these cosmological models.

Also one can consider the situation as in the present paper but with logarithmic corrections in SAdS₅ or in SdS₅ bulk backgrounds [32]; we hope to come back in the future to this important problem.

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